Inductance

Important fact: Magnetic Flux $\Phi_B$ is proportional to the current making the $\Phi_B$

All our equations for B-fields show that $B \propto I$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\vec{d}l \times \hat{r}}{r^2}$$

Biot-Savart

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{thru}$$

Ampere

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Inductance

Flux $\Phi_B \propto B \propto I$

Flux $\Phi_B \propto I$ Assume leaving everything else the same.

If we double the current $I$, we will double the magnetic flux through any surface.
Self-inductance

Self-Inductance (L) of a coil of wire

\[ \Phi_B \equiv LI \]

This equation defines self-inductance.

\[ L \equiv \frac{\Phi_B}{I} \]

Note that since \( \Phi_B \propto I \), L must be independent of the current I.

L has units \([L] = \text{[Tesla meter}^2]/\text{[Amperes]}\).

New unit for inductance = \([\text{Henry}]\).

Inductors (coil of wire)

An inductor is just a coil of wire.

The Magnetic Flux created by the coil, through the coil itself:

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

This is quite hard to calculate for a single loop. Earlier we calculated B at the center, but it varies over the area.
Inductors

Consider a simpler case of a solenoid

\[ |\vec{B}_{\text{inside}}| = \mu_0 n I = \mu_0 \frac{N}{L} I \]

Recall that the B-field inside a solenoid is uniform!

N = number of loops
L = length of solenoid
* Be careful with symbol L!

\[ \Phi_B = N \oint \vec{B} \cdot d\vec{A} = NBA = N(\mu_0 n I)A \]

\[ L = \frac{\Phi_B}{I} = \mu_0 NnA = \mu_0 An^2L \]

Self-Inductance
Length
Clicker Question

Inductor 1 consists of a single loop of wire. Inductor 2 is identical to 1 except it has two loops on top of each other. How do the self-inductances of the two loops compare?

A) $L_2 = 2L_1$  
B) $L_2 > 2L_1$  
C) $L_2 < 2L_1$

Answer: $L_2 > 2L_1$, in fact $L_2 = 4L_1$. The self inductance $L$ increases by 4. $L = \Phi/i$. If we keep $i$ fixed, but double the number $N$ of turns, $\Phi$ increases by 4. Total flux $N$ doubles, but $\Phi_1 = BA$ also doubles because when we double the number of turns, then $B$ is doubled.

Clicker Question

Two long solenoids, each of inductance $L$, are connected together to form a single very long solenoid of inductance $L_{\text{total}}$. What is $L_{\text{total}}$?

A) $2L$  
B) $4L$  
C) $8L$  
D) none of these/don't know

Answer: $2L$. The inductance of a solenoid is $L = \mu_0 n^2 A/I$, where $n$ is the number of turns per length $n = N/l$ and $l$ is the length. In this case, we did not change $n$, but $l$ (length) doubled, so $L$ doubles.
What does this inductance tell us?

\[ L = \frac{\Phi_B}{I} \]
\[ \Phi_B = LI \]

\[ \frac{d\Phi_B}{dt} = L \frac{dl}{dt} \]

L is independent of time. Depends only on geometry of inductor (like capacitance).

Recall Faraday’s Law

\[ \varepsilon = -\frac{d\Phi_B}{dt} \]

Changing the current in an inductor creates an EMF which opposes the change in the current. Sometimes called “back EMF”
Self-induced EMF

\[ \varepsilon = -L \frac{dI}{dt} \]

It is difficult (requires big external Voltage) to change quickly the current in an inductor.

The current in an inductor cannot change instantly.

If it did (or tried to), there would be an infinite back EMF. This infinite back EMF would be fighting the change!

Magnetic Field energy

Recall that for a capacitor C, there is stored potential energy in the electric field.

\[ U = \frac{1}{2} CV^2 \]

The energy is stored in the electric field and the density is:

\[ u_E = \frac{U}{Volume} = \frac{1}{2} \varepsilon_0 \mid \vec{E} \mid^2 \]
Magnetic Field energy

The power supplied to an inductor is

\[ P = Vi = Li \frac{di}{dt} \]

The energy \( dU \) supplied to an inductor in an interval \( dt \) is:

\[ dU = Pdt = Lidi \]

The total energy \( U \) supplied while the current increases from 0 to a final value is

\[ U = \int_0^i Lidi = \frac{1}{2} LI^2 \]

Magnetic energy density

For an inductor \( L \), with current \( I \), there is stored energy in the magnetic field.

\[ U = \frac{1}{2} LI^2 \]

For a coil this is

\[ U = \frac{1}{2} \frac{\mu_0 N^2 A}{2\pi r} I^2 \]

The energy density is

\[ u_B = \frac{U}{Volume} = \frac{U}{2\pi rA} = \frac{1}{2} \frac{\mu_0 N^2 I^2}{(2\pi r)^2} \]

But we know that

\[ B = \frac{\mu_0 NI}{2\pi r} \]

\[ \rightarrow u_B = \frac{B^2}{2\mu_0} \]
The same current $i$ is flowing through solenoid 1 and solenoid 2. Solenoid 2 is twice as long and has twice as many turns as solenoid 1, and has twice the diameter. (Hint) for a solenoid $B = \mu_0 n i$

What is the ratio of the magnetic energy contained in solenoid 2 to that in solenoid 1, that is, what is $\frac{U_2}{U_1}$?

A) 2  
B) 4  
C) 8  
D) 16  
E) None of these.

Answer: There is 8 times as much magnetic field energy in the large solenoid as in the small solenoid. The B-field is the same in both solenoids (same $n =$ turns/length so same $B = \mu_0 n I$) so both solenoids contain the same energy per volume $u = U/vol = B^2/(2\mu_0)$. The larger solenoid has 8 times the volume (2X the length, 4X the cross-sectional area) so it has 8 the energy.

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**Magnetic Field energy**

It takes work to get current flowing through an inductor.

You must work against the back EMF which opposes any change in the current.

That work = potential energy stored = $U = \frac{1}{2} LI^2$

And is thus stored in the inductor’s magnetic field.
Inductors as circuit elements

What do these inductors do in circuits?

Just recall that the EMF or Voltage across an inductor is:

$$\varepsilon = -L \frac{dl}{dt}$$

So, when we add them to circuits, we can apply the usual Kirchhoff’s Voltage Law and include the inductors.

Inductors in a circuit

Consider a circuit with a battery, resistor and inductor (RL circuit)

Switch is in position (a) for a long time.
In steady state (after a long time), the current will no longer be changing and thus the inductor looks like a regular wire!
If after a long time the inductor acts like a wire:

\[ \Delta V = V - IR = 0 \quad I = \frac{V}{R} \]

At t=0, move the switch to (b).

Normally one might expect there to immediately be zero current. However, the inductor has stored energy!
We need to solve this differential equation for $i(t)$.

**LR circuit**

\[ \Delta V = -IR - L \frac{dI}{dt} = 0 \]

\[ \frac{di}{dt} = -\left(\frac{R}{L}\right)i \]

**RL circuit**

\[ \frac{dI}{dt} = -\left(\frac{R}{L}\right)I \]

\[ I(t) = I_0 e^{-\left(\frac{R}{L}\right)t} \quad \text{where} \quad I_0 = \frac{V}{R} \]

\[ I(t) = I_0 e^{-\left(\frac{L}{R}\right)t} \quad \text{Current exponentially decays with Time Constant } = \tau = L/R \text{ (units of seconds)}. \]
Rank in order, from largest to smallest, the time constants \( \tau_1, \tau_2 \) and \( \tau_3 \) in the three circuits.

A. \( \tau_1 > \tau_2 > \tau_3 \)
B. \( \tau_2 > \tau_1 > \tau_3 \)
C. \( \tau_2 > \tau_3 > \tau_1 \)
D. \( \tau_3 > \tau_1 > \tau_2 \)
E. \( \tau_3 > \tau_2 > \tau_1 \)

\[ i(t) = i_0 e^{-t/(\tau R)} \]

The switch in the circuit below is closed at \( t=0 \).

What is the initial rate of change of current \( di/dt \) in the inductor, immediately after the switch is closed?
(Hint: what is the initial voltage across the inductor?)

A) 0 A/s     B) 0.5A/s     C) 1A/s     D) 10A/s     E) None of these.

Answer: 1A/s. By the Loop Law, \( V(\text{battery voltage}) = iR + Ldi/dt \). Immediately after the switch is closed, the current is zero (because the current thru the inductor cannot change instantly), so \( iR \) is zero, so \( V = L \text{ di/dt} \). So \( di/dt = V/L = 10V/10H = 1 \text{ A/s} \).
An LR circuit is shown below. Initially the switch is open. At time t=0, the switch is closed. What is the current thru the inductor L immediately after the switch is closed (time = 0+)?

A) Zero  
B) 1 A  
C) 0.5A  
D) None of these.

Clicker Question

After a long time, what is the current from the battery?
A) 0A  
B) 0.5A  
C) 1.0A  
D) 2.0A  
E) None of these.
Dual Power - Best Price On the Net!!! (don't pay $20 or more) - Super Bright Crystal White Light - Classic Focused Beam - Never Needs Batteries!

This is the BEST LED Shaker Light! Bright, Compact, Dual Power!

The Shaker LED Flashlight is dual powered! This gives you 12+ hours of portable battery powered light. When the batteries run down in 99%, it switches to the Shake & Shine Power!

How it Works: The LED Shaker Light uses a dual power system:
1) Works on two CR-2032 batteries which last well over 12 hours of constant usable light. The batteries are replaceable like a regular flash light.
2) Works on Faraday’s Principle of Magnetic Induction to produce a bright Crystal White light without batteries. As the flashlight is shaken, a magnet passes through a metal coil which, when you shake, creates a voltage in the coil generating electricity. During prolonged use it can be shaken for 10-15 seconds every 2 or 3 minutes for bright light.

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