So far we have studied two forces: **gravity and electricity**

Magnetism is a new force, but also related to electric charges.

Gravity is created by mass and gravity acts on masses.

Electric fields are created by electric charges \( E = \frac{kQ}{r^2} \hat{r} \)

And Electric fields exert forces on charges. \( \vec{F}_E = q\vec{E} \)

Magnetic field and force

There is a different kind of field, called a magnetic field, or B-field

B-fields are created by **moving charges** (currents).

B-fields exert a force on **moving charges**.

This is very different from our previously studied forces!
Natural magnetism

(a) Opposite poles attract.

(b) Like poles repel.

Natural magnetism

The geomagnetic north pole is actually a magnetic south (S) pole—it attracts the N pole of a compass.

Compass

Magnetic field lines show the direction a compass would point at a given location.

The earth’s magnetic field has a shape similar to that produced by a simple bar magnet (although actually it is caused by electric currents in the core).

The earth’s magnetic axis is offset from its geographic axis.

North geographic pole (earth’s rotation axis)

South geographic pole
What is the origin of this force?

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.

Breaking a magnet in two ...

... yields two magnets, not two isolated poles.

Magnetic charge? Monopoles?

Scientists have postulated that there might be some other kind of object that has a “magnetic charge” = Magnetic Monopole.

No magnetic monopoles have ever been found.

Thus, we do not consider them in classes.
Magnetism and electric charge

Oersted’s experiment:
A compass is placed directly over a wire (here viewed from above). A passing current deflects the needle.

Force on a moving charge

We will see later exactly how B-fields are made by moving charges.

Now we will study how B-fields exert forces on moving charges.

\[ \vec{F} = q\vec{v} \times \vec{B} \]

The magnetic force exerted on a charge q moving with velocity v in a magnetic field B.
This equation actually defines the magnetic B-field.
Visualizing a M. Field: Field lines

http://www.youtube.com/watch?v=wuA-dkKvrd0

Force on a moving charge

\[ \vec{F} = q \vec{v} \times \vec{B} \]

Vector cross product.

\[ |\vec{F}| = q |\vec{v}| |\vec{B}| \sin \theta \]

What about the resulting force direction from the cross product?
Vector cross product – Right Hand Rule

-$\vec{u} \times \vec{v}$

- Fingers in direction of first vector.
- Bend them into direction of second vector.
- Thumb points in cross product direction.
- If your hand does not bend that way, flip it around! – never use the left hand ;-) 

New vector direction is always perpendicular to the original vectors!

Force on a moving charge

(a) Right-hand rule for the direction of magnetic force on a positive charge moving in a magnetic field:
1. Place the $\vec{v}$ and $\vec{B}$ vectors tail to tail.
2. Imagine turning $\vec{v}$ toward $\vec{B}$ in the $\vec{v}$-$\vec{B}$ plane (through the smaller angle).
3. The force acts along a line perpendicular to the $\vec{v}$-$\vec{B}$ plane. Curl the fingers of your right hand around this line in the same direction you rotated $\vec{v}$. Your thumb now points in the direction the force acts.

(b) If the charge is negative, the direction of the force is opposite to that given by the right-hand rule.
1. $\vec{F} = q\vec{v} \times \vec{B}$
2. $\vec{F} = -q\vec{v} \times \vec{B}$
Consequences:

\[ \vec{F} = q \vec{v} \times \vec{B} \]

\[ | \vec{F} | = q | \vec{v} | | \vec{B} | \sin \theta \]

- If particle is not moving (v=0) then no force.
- If velocity and B-field are parallel, then no force.
- If velocity and B-field are perpendicular, then maximum force.
- If q is negative, then force is in opposite direction.

Drawing conventions

Drawing vector directions on a 2 dimensional piece of paper?

- Magnetic Field Vector to the right.
- Magnetic Field Vector out of the page.  
  **Arrow head pointing at you.**
- Magnetic Field Vector into the page.  
  **Arrow tail pointing at you.**
Clicker Question
A negative particle and a positive particle are moving with certain velocities in a constant, uniform magnetic field, as shown. The direction of the B-field is to the right. The (+) particle is moving directly left; the (−) particle is moving directly up.

The force on the positive particle due to the B-field is (in = into page, out = out of page).

A: in  B: out  C: zero  D: right  E: left

Answer: The (+) particle is moving anti-parallel to the B-field. The angle θ is 180 and the force is \(F_B = qvB \sin \theta = 0\).

Clicker Question
A negative particle and a positive particle are moving with certain velocities in a constant, uniform magnetic field, as shown. The direction of the B-field is to the right. The (+) particle is moving directly left; the (−) particle is moving directly up.

The force on the negative particle due to the B-field is

A: in  B: out  C: zero  D: right  E: left

Answer: The (−) particle is moving at right angles to the field. By the right-hand rule, the direction "\(v \times B\)" is into the page, but the particle has a negative charge \(q\), so the force is out of the page.
Clicker Question

A positive particle is released from rest in a region of space where there is constant, uniform, electric field and a constant, uniform magnetic field. The electric field points up and the magnetic field points out of the page in the diagram below. Which path will the positive particle follow? (All paths shown are in plane of the page.)

Answer: The (+) particle will feel a force $F_E = qE$ due to the $E$-field along the direction of the $E$-field. As it starts moving along the $E$-field direction, it will acquire a velocity, and it will start to feel a force $F_B=qvB$, due to the $B$-field. The direction of the force is to the right, by the right-hand-rule.

Clicker Question

A negative particle and a positive particle are moving with certain velocities in a constant, uniform magnetic field, as shown. The direction of the $B$-field is to the right. The (+) particle is moving directly left; the (–) particle is moving directly up.

The force on the negative particle due to the $B$-field is

A: in B: out C: zero
D: right E: left

Answer: The (–) particle is moving at right angles to the field. By the right-hand rule, the direction "$v$ cross $B$" is into the page, but the particle has a negative charge $q$, so the force is out of the page.
Magnetic field units

Units for Magnetic Field

\[ \vec{F} = q \vec{v} \times \vec{B} \]

B-field = [B] = [Newton] / [Coulomb x meters/second] = [Tesla]

How big is a 1 Tesla Magnetic Field?

- Interstellar Space: $10^{-10}$ Tesla
- Human Being: $10^{-10}$ Tesla
- Earth’s Surface: $5 \times 10^{-5}$ Tesla
- Sun’s Surface: $10^{-2}$ Tesla
- Small Bar Magnet: $10^{-2}$ Tesla
- Experiment Magnet: 1 Tesla
- Maximum Steady Magnet: 30 Tesla
- Maximum in Explosive Magnet: 1000 Tesla
- Surface of Neutron Star: $10^8$ Tesla
Another Unit System

1 Gauss = $10^{-4}$ Tesla

Thus, the Earth’s magnetic field is
~ 0.5 Gauss.

This unit system is often used when talking about small magnetic fields, but it is **not the SI unit system!**

Earth’s magnetic field
Clicker Question

Here is an event display from a high energy experiment. There is a 1 Tesla uniform magnetic field coming out of the page. What is the sign of the electric charge?

A) Positive  
B) Negative

\[ \vec{F} = q\vec{v} \times \vec{B} \]

Videos

Feynman's "why" and "Fields"
Motion of charged particles in a Magnetic Field

Because the force is always perpendicular to the velocity (direction of motion), the Magnetic force can do no work on q.

\[ \vec{F} = q\vec{v} \times \vec{B} \]

\[ W = \vec{F} \cdot \Delta \vec{r} = 0 \]

since \( \vec{F} \perp \Delta \vec{r} \)

B-field cannot change the Kinetic Energy of a moving particle, but can change its direction of motion.

\[ W_{net} = \Delta KE = 0 \]

Charged particle in a perpendicular field

Particle moving in a plane with a B-field uniformly out of the plane.

Results in circular motion!

No change in KE, but constant change in velocity direction.
Charged particle in a perpendicular field

\[ |\vec{F}| = m |\vec{a}| = q \vec{v} \times \vec{B} \]

\[ |\vec{F}| = q |\vec{v}| |\vec{B}| \quad \text{Since velocity and } B \text{ are always perpendicular.} \]

\[ |\vec{F}| = q |\vec{v}| |\vec{B}| = \frac{mv^2}{R} \quad \text{Since circular motion.} \]

\[ R = \frac{mv}{qB} \]

Radius of circular motion depends on m, v, q, B.

Charged particle in a perpendicular field

One can then solve for the frequency of revolution ("cyclotron frequency").

\[ f = \text{# revolutions/second} \]

\[ v = \frac{qRB}{m} = \frac{\text{distance}}{\text{time}} = \frac{2\pi R}{T} \quad \text{Period} \]

\[ f = \frac{1}{T} = \frac{qB}{2\pi m} \]

The cyclotron frequency is independent of R.
Helical motion

What if in addition to the circular motion in this plane, there is a non-zero velocity out of the page?

This extra velocity contributes no additional force since it is parallel to $B$.

$$R = \frac{mv}{qB}$$

Aurora borealis (“Northern lights”)

Van Allen radiation belts

Particile motion is spiral or helical.
Clicker Question

A (+) charged particle with an initial speed $v_0$ is moving in a plane perpendicular to a uniform magnetic field (B into the page). There is a tenuous gas throughout the region which causes viscous drag and slows the particle over time. The path of the particle is

A: a spiral inward
B: a spiral outward
C: something else

Applications: Mass spectrometer
Applications: velocity selector

(a) Schematic diagram of velocity selector

- Source of charged particles
- Electric field $E$
- Magnetic field $B$
- Force on a positive charge $q$:
  - By the right-hand rule, the force of the $B$ field on the charge points to the right.
  - The force of the $E$ field on the charge points to the left.
- For a negative charge, the directions of both forces are reversed.

(b) Free-body diagram for a positive particle

- Electric force: $F_E = qE$
- Magnetic force: $F_B = qvB$
- Total force: $\sum F = qvB - qE$

For the particle to pass and not be deflected:

$$qvB - qE = 0 \rightarrow v = \frac{E}{B}$$

Thomson’s $e/m$ experiment

- Electrons travel from the cathode to the screen.
- Between plates $P$ and $F$, they are mutually perpendicular, uniform $E$ and $B$ fields.

From energy conservation:

$$\frac{1}{2}mv^2 = eV \rightarrow v = \sqrt{\frac{2eV}{m}}$$

We have seen that to pass through we need:

$$v = \frac{E}{B}$$

Combining the two we obtain:

$$\frac{e}{m} = \frac{E^2}{2VB^2}$$
Force on current carrying wires

Since B-fields exert forces on moving charges, it is natural that B-fields exert forces on current carrying wires.

How do we quantify this force in terms of current $I$, instead of $q$ and $v$?

Consider the wire shown below. Now add a Magnetic Field.

$\vec{F} = q\vec{v} \times \vec{B}$

$\vec{F}$ (one charge) = $q\vec{v} \times \vec{B}$

# charges in this segment of wire

$N = \frac{n \times A \times L}{\text{Volume}}$

$\vec{F}$ (total) = $(nALq)\vec{v} \times \vec{B}$
Force on current carrying wires

\[ \vec{F}_{\text{total}} = (nALq)\vec{v} \times \vec{B} \]

How to relate this to current?

\[ J = \frac{I}{A} = nqv_d \]

\[ I = nAqv_d \]

\[ \vec{F}_{\text{total}} = I\vec{L} \times \vec{B} \]

* Notice that \( \vec{L} \) points along \( \vec{v} \), which is the direction of the current.

Clicker Question

A current-carrying wire is in a B-field. The wire is oriented to the B-field as shown. What is the direction of the magnetic force on the wire?

A) Right
B) Down
C) Out of the Page
D) Into the Page
E) None of these.
Force on a straight wire

\[ \vec{F}(total) = i\vec{L} \times \vec{B} \]

Force on electric wires due to Earth’s Magnetic Field

Power line of 1000 meters runs along the Earth’s equator where the B-field = 0.5 Gauss points South to North.

The current in the wire is 500 Amps going East to West.

\[ \vec{F}(total) = i\vec{L} \times \vec{B} \]
\[ \vec{F} = (500A)(1000m)(0.5 \times 10^{-4}T) = 25N \text{ up} \]

Weight of the wire ~ 20,000 Newtons.
General expression for the force on a wire

If the wire is not straight or the B-field is not uniform, we need to break the wire up into little segments

\[ d\vec{F} = i d\vec{L} \times \vec{B} \]

\[ \vec{F}_{tot} = \int d\vec{F} = \int i d\vec{L} \times \vec{B} \]

Clicker Question

A square loop of wire carrying current I is in a uniform magnetic field B. The loop is perpendicular to B (B out of the page). What is the direction of the net force on the wire?

A: out of the page
B: into the page
C: ↑
D: →
E: None of these
The DC motor

We can see (Section 27.7) that a magnetic field can produce \textit{torque} on a loop of wire carrying a current.

(a) Brushes are aligned with commutator segments.
- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

(b) Rotor has turned 90°.
- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

(c) Rotor has turned 180°.
- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

Magnetic Field Lines and Flux

(a) Magnetic field of a C-shaped magnet
Between flat, parallel magnetic poles, the magnetic field is nearly uniform.

(b) Magnetic field of a straight current-carrying wire
To represent a field coming out of or going into the plane of the paper, we use dots and crosses, respectively.

(c) Magnetic fields of a current-carrying loop and a current-carrying coil (solenoid)
Notice that the field of the loop and, especially, that of the coil look like the field of a bar magnet (see Fig. 27.11).
Gauss’s Law for Magnetic Fields

Magnetic monopoles (so far) do not exist!!!
There are no sources of magnetic flux

=> Flux through a *closed* surface:
\[ \int \vec{B} \cdot d\vec{A} = 0 \]