RC circuits

Initially one has $+Q_0$ and $-Q_0$ on the Capacitor plates. Thus, the initial Voltage on the Capacitor $V_0 = Q_0/C$. What do you think happens when the switch is closed?

\[ \text{Discharging a capacitor} \]

Close the switch at $t=0$, then current $i$ starts to flow.

At $t=0$, \[ i_0 = \frac{V_0}{R} \]

Later \[ i(t) = -\frac{dQ}{dt} \]

* Negative sign since $Q$ is decreasing.
Discharging a capacitor

Voltage across $C$ = Voltage across $R$

\[ V_c = V_R \]

\[ \frac{Q}{C} = iR = -\frac{dQ}{dt}R \]

\[ \frac{dQ}{dt} = -\frac{1}{RC}Q \]

We now need to solve this differential equation.

Solving the differential equation:

\[ \frac{dQ}{dt} = -\frac{1}{RC}Q \]

\[ Q(t) = Q_0 e^{-t/RC} \]

Check solution by taking the derivative...

\[ \frac{dQ}{dt} = Q_0 \left( -\frac{1}{RC} \right) e^{-t/RC} = -\frac{1}{RC}Q \]

Also $Q(t) = Q_0$ at $t=0$.

\[ Q(t = 0) = Q_0 e^{-0/RC} = Q_0 \]
Exponential Decay

\[ Q(t) = Q_0 e^{-t/RC} \]

After a time \( \tau = RC \), \( Q \) has dropped by \( e^{-1} = 1/e \).

After a time \( t = 2RC \), \( Q \) has dropped by \( e^{-2} = 1/e^2 \).

Thus \( \tau = RC \) is often called the time constant and has units [seconds].

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Discharging a capacitor

\[ \frac{dQ}{dt} = Q_0 \left( -\frac{1}{RC} \right) e^{-t/RC} = -\frac{1}{RC} Q \]

\[ |i(t)| = \left| \frac{dQ}{dt} \right| = \left( \frac{Q_0}{RC} \right) e^{-t/RC} = i_0 e^{-t/RC} \]

Thus, the current also falls with the same exponential function.
Clicker Question

A capacitor with capacitance 0.1F in an RC circuit is initially charged up to an initial voltage of $V_o = 10V$ and is then discharged through an $R=10\Omega$ resistor as shown. The switch is closed at time $t=0$. Immediately after the switch is closed, the initial current is $I_o = V_o / R = 10V / 10\Omega$.

What is the current $I$ through the resistor at time $t=2.0$ s?

A) 1A  
B) 0.5A  
C) $1/e$ A = 0.37A  
D) $1/e^2$ A = 0.14A  
E) None of these.

Answer: $1/e^2$ A = 0.14A.

The time constant for this circuit is $RC = (10\Omega)(0.10F) = 1.0$ sec. So at time $t=2.0$ sec, two time constants have passed. After one time constant, the voltage, charge, and current have all decreased by a factor of $e$. After two time constants, everything has fallen by $e^2$. The initial current is 1A. So after two time constants, the current is $1/e^2$ A = 0.135A.

Charging a capacitor

More complex RC circuit: Charging C with a battery.

Before switch closed $i=0$, and charge on capacitor $Q=0$.

Close switch at $t=0$.

Try Voltage loop rule.

\[ +V_b + V_R + V_C = 0 \]
\[ +V_b - iR - Q / C = 0 \]
Charging a capacitor

\[ +V_b - iR - \frac{Q}{C} = 0 \]

\[ +V_b - \frac{dQ}{dt} \frac{R}{C} = 0 \]

\[ \frac{dQ}{dt} = + \frac{V_b}{R} - \frac{Q}{RC} \]

\[ Q(t) = CV_b \left(1 - e^{-t/RC} \right) \]

- Although no charge actually passes between the capacitor plates, it acts just like a current is flowing through it.
- Uncharged capacitors act like a "short": \( V_c = Q/C = 0 \)
- Fully charged capacitors act like an "open circuit". Must have \( i_c = 0 \) eventually, otherwise \( Q \to \) infinity.
An RC circuit is shown below. Initially the switch is open and the capacitor has no charge. At time $t=0$, the switch is closed. What is the voltage across the capacitor immediately after the switch is closed ($time = 0$)?

A) Zero  
B) 10 V  
C) 5V  
D) None of these.