How do we charge a capacitor?

Battery
Device that generates and maintains an electrical potential difference.

People knew how to make a brief spark, but not how to keep charges moving.

Volta invented the battery in 1800’s.

Two metal plates are called electrodes.

Liquid or Paste inside battery called electrolyte (often a strong acid)
Battery

By using two different metals for the electrodes, we can have two different chemical reactions.

The reactions build up charges creating the electrical potential difference between the metal electrodes.

One unit is called a “cell”.
Many cells put together is called a battery.

Example: Car battery is six 2 V cells = 12 V battery

Each cell consists of a lead (Pb) electrode and a lead oxide (PbO₂) electrode immersed in a solution of water and sulfuric acid (H₂SO₄).

The “Mac Gyver” Battery
When the battery is not connected, the charges have nowhere to go; they feel a potential, but they stay in the electrodes. When we “close” the circuit, charges can move, can circulate, producing a “current.”

Electro motive force (EMF)

Analog between a battery and an escalator.

http://www.youtube.com/watch?v=F1p3fgbDnkY&NR=1
Charging a capacitor

The electrons will flow through the wires. However, they cannot jump through the empty space between the plates. Charges will accumulate in the plates until the potential difference or voltage equals the one in the battery.

Electric current

\[ I \equiv \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \quad \text{Rate of flow of charge} \]

Units \([I]\) = Coulombs/second = Ampere (A) or “Amp”
What or “who” is carrying the current?

Electrons flow in metals, not the protons, so the negative electric charges are moving.

\[ \vec{F} = q\vec{E} = -e\vec{E} \]

Electrons go “upstream” against the electric field vector.

Flow of negative charge in one direction is equivalent to flow of positive charge in the opposite direction.

Current direction

By convention, we define the direction of current “\( I \)” as the direction of flow of (imaginary) positive charges.

In reality, of course, it is often negative charges flowing the other way.

Whose fault is this?
Microscopic description

\[ dQ = q(dN) = q(nAv_d dt) = nqAv_d dt \]

\[ \rightarrow I = \frac{dQ}{dt} = nqAv_d = JA \]

\[ J = \frac{I}{A} = nqv_d \]

Example

How many electrons are flowing by when \( I = 1.0 \text{ A} \)?

\[ I = \frac{\Delta Q}{\Delta t} = e \frac{\Delta N_e}{\Delta t} \]

\[ e = 1.6 \times 10^{-19} \text{ C} \]

\[ \frac{\Delta N_e}{\Delta t} = \frac{I}{e} = 5.6 \times 10^{18} \text{ elec/sec} \]

\( I = 10 \text{ mA} = 0.01 \text{ Amps} \) is lethal!

Yet, I could grab a wire carrying 1000 Amps and be safe.

Why? My body has a much higher electrical resistance than metal. Thus, the electrons prefer to mostly flow through the wire – not my body!
Electric resistance

Most materials are **Ohmic**.

In Ohmic materials, the current (I) is proportional to the electric field or the electrical potential difference (V).

\[ V = \Delta V = |\bar{E}|d \]
\[ I \propto V \quad \text{(Ohmic material)} \]
\[ \frac{V}{I} = \text{constant} \]

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Ohm’s Law

Resistance R of a piece of wire (or other material)

\[ R \equiv \frac{V}{I} = \text{constant} \]

\[ V = IR \]

“Ohm’s Law”

This is not really a physical law.
It is true for perfectly Ohmic materials.

Resistance has units of Ohms = \( \Omega \) = Volts/Amp
CPS question

A light bulb is attached to a battery with constant voltage $V$. The light filament has resistance $R$. When the light bulb is first turned on by attaching to the battery, the filament heats up rapidly, and as it heats, its resistance $R$ increases (due to increased scattering of electrons by thermal vibrations). As the light bulb filament heats up, the current $i$ in the filament

A) Increases
B) Decreases
C) Remains the same.

V stays constant, $R$ increases, so $I = \frac{V}{R}$ decreases

CPS question

An ideal battery produces a fixed?

A) Current Output
B) Electric Potential Energy Output
C) Power Output
D) Electric Potential Difference
E) None of the Above
Ohmic behavior

Silicon chip characteristic.
Where is the Ohmic and non Ohmic behavior?

Resistors

Ideal wire \( R_{\text{wire}} = 0 \).

\[ \Delta V_{\text{wire}} = IR_{\text{wire}} = 0 \]

Ideal battery \( R_{\text{battery}} = 0 \).
Resistors (cont.)

Flow of electrons in wire or resistor is like flow of water in a full pipe (no bubbles or leaks).

A \quad C \quad B

\[ I_A = 2 \text{ gallons/min} \quad I_B = 2 \text{ gal/min} \quad I_C = 2 \text{ gal/min} \]

High Pressure

A \quad B \quad C

Low Pressure

High Voltage

I \quad \quad I \quad \quad I

Low Voltage

Resistivity

Resistance R of a piece of conductor depends on
1) Composition or Material
2) Shape and Dimensions

\[ R = \rho \frac{L}{A} \]

Where \( \rho \) is resistivity – measure of internal friction; dependent on material composition.
What is the source of resistivity?

Resistivity originates from “internal friction” or interaction between the electrons carrying the current and other degrees of freedom inside the solid, mainly atoms vibrating (phonons).

Resistivity and temperature

\[ \rho(T) = \rho_0 [1 + \alpha(T - T_0)] \]

Temperature dependence over a small interval in \( T \)

(Taylor expansion)

\( \alpha \) is called “temperature coefficient of resistivity”
CPS question

Two cylindrical resistors are made of the same material (same resistivity $\rho$). Resistor 2 is twice as long and has twice the diameter of resistor 1. What is the ratio $R_2/R_1$?

A) 2  B) 4  C) 1/2  D) 1/4  E) 1

Answer:

$$\frac{R_2}{R_1} = \frac{L_2}{L_1} \frac{A_1}{A_2} = 2 \times \frac{1}{4} = \frac{1}{2}$$

(The resistivities cancel)

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Microscopic version of Ohm’s Law

$$J = \frac{I}{A} = \frac{\text{current}}{\text{area}}$$

Current density

$$V = IR = (JA) \left( \frac{\rho L}{A} \right) = J\rho L$$

$$V = EL$$

They are equivalent.

http://www.youtube.com/watch?v=5laTkjJNHrg&feature=related
Conductivity

\[ J = \frac{1}{\rho} E = \sigma E \]

Conductivity is the inverse of resistivity.

Same as resistivity, depends only on the composition or material

CPS question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.

Which region A, B, or C has the greatest magnitude current I?
A  B  C  D) all three have the same I
CPS Question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.

Which region A, B, or C has the greatest magnitude current density J?
A) B C D) all three have the same J.

Region B has the largest current density $J = I/A$. All 3 regions have the same current $I$, so the region with the smallest $A$ has the largest $J$.

CPS question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.

Which region A, B, or C has the greater conductivity?
A) B C D) all three have the same $\sigma$.

All three regions have the same conductivity $\sigma$. Conductivity $\sigma$ is a property of the material type, not of the shape of the sample.
CPS question

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.

Which region A, B, or C has the greatest magnitude electric field?
A B C D) all three have the same $|E|$

Region B has the largest E-field. $J = \sigma E$, $J$ is largest is region B (from the 1st part of this question), $\sigma$ is constant for all regions, so $E$ must be max where $J$ is max, in region B.

Potential changes around a circuit
Energy and Power in circuits

What happens to the resistor (light bulb) when we turn it on?

It emits light. It also gets hot! **Heat is a form of energy.**

Power \( P = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Joules}}{\text{second}} = \text{Watts} \)

Inside a current carrying resistor, electrostatic potential energy is converted into thermal energy (heat).

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Energy and Power in circuits

The moving charges (for example electrons) are shifting down in potential energy. In an interval of time \( \Delta t \), the amount of charge passing through a circuit element is:

\[ \Delta Q = I \Delta t \]

The potential energy change for this amount of charge is

\[ \Delta U = (\Delta Q)V = (I\Delta t)V \]

The power delivered or extracted from the circuit element is:

\[ P = \frac{\Delta U}{\Delta t} = \frac{I(\Delta t)V}{\Delta t} = IV \]
Power delivered to a resistor

\[ P = IV \]

If the resistor is Ohmic, then \( V = IR \) and thus

\[ P = IV = I^2R = \frac{V^2}{R} \]

Where did the energy go?
Energy goes into heat. Heat is in fact random (thermal) motion of particles, which is microscopic kinetic energy. However, these velocities are random and do not change the overall drift velocity.

Clicker Question

“100 Watt” light bulb
For all household appliances, they have:

\[ \Delta V = 120V \]

What is the approximate resistance of the filament (at operating temperature)?
A) \( R = 100 \text{ Ohms} \)
B) \( R = 144 \text{ Ohms} \)
C) \( R = 1250 \text{ Ohms} \)
D) None of the Above

In fact, 95% is heat, and only 5% of energy is in the light.