Electric field lines

Imaginary lines or curves drawn through a region of space such that their tangent at any point are in the direction of the electric field vector at that point.

Electric field lines (cont.)

- Higher density of lines $\rightarrow$ stronger field
- Arrows define the direction of the field
- Field lines never intersect (the field is uniquely defined at each point)
How many field lines cross out of the circle?

- \(8 \text{C} \Rightarrow 8 \text{ lines}\)
- \(16 \text{C} \Rightarrow 16 \text{ lines}\)
- \(32 \text{C} \Rightarrow 32 \text{ lines}\)
Lines of Electric Field

How many field lines cross out of the surface?

8C ⇒ 8 lines
16C ⇒ 16 lines
32C ⇒ 32 lines

Lines of Electric Field

How many field lines cross out of the surface?

ZERO!!!
Gauss’s law

Number of lines crossing the closed surface: 0!!!

Observations

• Charges outside the surface do not contribute to the sum
• The number of lines crossing the surface is proportional to the net amount of charge inside
• The number of crossing lines is independent of the shape of the surface

http://www.youtube.com/watch?v=5ENl4vn82bc&NR=1
http://webphysics.davidson.edu/physlet_resources/bu_semester2/index.html
Gauss’s Law: Cartoon Version

The number of electric field lines leaving a closed surface is equal to the charge enclosed by that surface.

\[ \Sigma (\text{E-field-lines}) \propto \text{Charge Enclosed} \]

- N Coulombs \( \Rightarrow \alpha N \) lines

Flux of a uniform field

(a) Surface is face-on to electric field:
- \( \vec{E} \) and \( \vec{A} \) are parallel (the angle between \( \vec{E} \) and \( \vec{A} \) is \( \phi = 0 \)).
- The flux \( \Phi_E = \vec{E} \cdot \vec{A} = EA \).

(b) Surface is tilted from a face-on orientation by an angle \( \phi \):
- The angle between \( \vec{E} \) and \( \vec{A} \) is \( \phi \).
- The flux \( \Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi \).

(c) Surface is edge-on to electric field:
- \( \vec{E} \) and \( \vec{A} \) are perpendicular (the angle between \( \vec{E} \) and \( \vec{A} \) is \( \phi = 90^\circ \)).
- The flux \( \Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90^\circ = 0 \).

\[ \Phi_E = E \perp A = EA \cos \varphi = \vec{E} \cdot \vec{A} = (\vec{E} \cdot \hat{n}) A \]
General expression for electric flux

For an arbitrary surface, take the component of $E$ perpendicular to the surface at that point, $E_\perp$, and integrate over the surface

$$\Phi_E = \oint E \cdot d\vec{A} = \oint \vec{E} \cdot \hat{n} dA = \oint E_\perp \cdot dA$$

**Flux through a cube**

(a) (b)
Flux through a sphere

The field is always perpendicular to the surface:

\[ \Phi_E = EA = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} (4\pi R^2) = \frac{q}{\varepsilon_0} \]

Two spheres with different radii

The same number of field lines and the same flux pass through both of these area elements.

\[ \Phi_E = EA = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} (4\pi R^2) = \frac{q}{\varepsilon_0} \]

The flux does not depend on the area, only on the charge enclosed by it!!!
Gauss’s Law – The Idea

A point charge outside a closed surface that encloses no charge: If an electric field line from the charge enters the surface at one point, it must leave at another.

Charges are “sources” of flux: electric lines can only begin or end inside a region of space only when there is a charge inside.

Gauss’s Law – The Idea

The total “electric flux” through any of these surfaces is the same and depends only on the amount of charge inside.
General form of Gauss’ law

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \]

(a) The outward normal to the surface makes an angle \( \phi \) with the direction of \( \vec{E} \).

(b) The projection of the area element \( d\vec{A} \) onto the spherical surface is \( dA \cos \phi \).